Economic load dispatch (ELD) is one of the most important optimization problems in the modern power system. The introduction of non-convex, non-differentiable and non-continuous models like valve point loading (VPL) and prohibited operating zone (POZ) makes the conventional ELD problem to a highly non-linear constrained problem which makes the conventional method to stick to local optima. In this paper, grey wolf optimization (GWO) algorithm which inherits the social and hunting behavior of grey wolves is used to solve such non-linear, non-convex ELD problem. The effectiveness of the GWO algorithm is verified by testing it on two ELD problems with VPL. The performance of GWO algorithm is validated using the statistical measures like minimum, maximum, mean and standard deviation over 50 independent test runs. Comparative results reveal that GWO algorithm for the chosen non-linear ELD problem performs better in terms of solution quality and robustness.

**KEYWORDS**: Economic Load Dispatch (ELD), Valve point loading (VPL), Prohibited Operating Zones (POZ), Ramp rate, Grey Wolf Optimization (GWO).

### 1. INTRODUCTION

In power generation system, economic load dispatch (ELD) problem is crucial for real-time modern power system operation and control. The ELD problem allocates the total electricity generation among the available thermal power generating units by satisfying all the constraints. Mathematical assumptions like convex, quadratic, differentiable and linear objectives are introduced in the classical ELD problem to solve it using conventional methods like lambda-iteration, gradient, dynamic programming etc [1][2].

However, in practical, the classical ELD problem is represented as a non-smooth or non-convex optimization problem which makes it difficult for the traditional methods to obtain the global optimum due to the nonlinearities and discontinuities like valve point loading (VPL), ramp rate limits, prohibited operating zones (POZ) and multi-fuel options. In addition, this non-linear ELD problem has a number of local optima and multiple constraints, which makes difficult for the traditional optimization techniques to solve as these methods usually tend to converge to a local optimum and suffer from "curse of dimensionality" [3].

Considering the limitations of the traditional optimization algorithms in solving these ELD problems with large dimensional and discrete search space, many modern meta-heuristic optimization algorithms have been proposed by various researchers and has solved the ELD problem with good convergence characteristics, high solution quality and robustness by eliminating the limitations of traditional methods[4][5].

Grey Wolf Optimization (GWO) algorithm proposed by Mirjalili et al is a recent swarm intelligence meta heuristics algorithm to solve the non-convex optimization problem [6]. The leadership and hunting behaviors of grey wolves which has superior exploration and exploitation ability is impersonated in GWO algorithm. With few parameters and ease of implementation, the GWO algorithm has the capability of providing higher quality solutions and good computational efficiency in solving real world problems [4][6]. These properties have motivated few researchers to implement the GWO algorithm in many problems like combined heat and power dispatch, hyper spectral band selection, load frequency control [8], optimal reactive power dispatch, power system stabilizer design, MPPT design, flow shop scheduling, attribute reduction, feature selection, parameter estimation and automatic generation control [7][4]-[16]. In this paper, GWO algorithm is used to solve the ELD problem with VPL to validate the effectiveness of GWO algorithm over other meta heuristics algorithms. The simulation results show that the GWO algorithm performs better than the previous algorithms in terms of solution quality, convergence efficiency and robustness.

Section 2 describes the formulation of ELD problem with VPL and constraints like ramp rate limits and POZ. Section 3 gives the detailed description of GWO algorithm. Implementation of GWO to the complex ELD problem is described in Section 4. The numerical results and discussion of the proposed algorithm for various test systems are presented in Section 5 and conclusion is drawn in Section 6.

### 2. FORMULATION OF ELD PROBLEM WITH VPL

#### 2.1 Objective function

![Fig. 1 Input - output curve of the generator with VPL](image)

The valve point effects which occur in the generators with multiple valve steam turbines as each steam admission valve in a turbine, produces a rippling effect on the unit’s cost curve and which cannot be represented by the simple polynomial function. Hence, in [17][18], the VPL has been modeled as a...
recurring rectified sinusoidal function as shown in Fig. 1.

Now, the objective of the constrained optimization ELD problem with VPL is formulated as

$$\text{Minimize } F_T = \sum_{i=1}^{N} F_i(P_i)$$ (1)

where the cost function of $i^{th}$ generator $F_i(P_i)$ is expressed as a summation of sinusoidal function and quadratic polynomial as given by

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i + |e_i \times \sin(f_i \times (P_{i,\text{min}} - P_i))|$$ (2)

where $a_i, b_i, c_i, e_i$ and $f_i$ are the cost coefficients of $i^{th}$ generator.

The above objective function of the ELD problem with VPL is minimized with the constraints given in Eqn. (3) to Eqn. (5).

2.2 Real Power balance or demand constraint

$$\sum_{i=1}^{N} P_i = P_D$$ (3)

where $P_D$ is the total power demand of the system (MW)

2.3 Real power generating limits

$$P_{i,\text{min}} \leq P_i \leq P_{i,\text{max}}$$ (4)

where $P_{i,\text{min}}$ and $P_{i,\text{max}}$ are the minimum and maximum limits of the generator $i$ in MW.

2.4 Ramp rate limits

Theoretically the assumption of smooth and instantaneous adjustment of generator output is considered in classical ELD problem. But in practical, it is implausible as the ramp rate limits of each generators i.e., both the up-rate limit $UR_i$ and down-rate limit $DR_i$ and previous hour generations $P_i^0$ are considered. Hence, for generator $i$,

$$P_i - P_i^0 \leq UR_i$$, if the power generated from unit $i$ increases

$$P_i^0 - P_i \leq DR_i$$, if the power generated from unit $i$ decreases

Therefore, using the ramp rate limits the real power generating constraints given in Eqn. (4) can be modified as

$$\text{Max} (P_{i,\text{min}}, P_i - P_i^0 - DR_i) \leq P_i \leq \text{Min} (P_{i,\text{max}}, P_i^0 + DR_i)$$ (5)

3. Grey wolf optimization (GWO) algorithm

A very recent meta-heuristic optimization Grey wolf optimization (GWO) algorithm inspired by grey wolves is developed and proposed by Mirjalili et. al. The GWO algorithm imitates the hunting and the social hierarchy behaviors of grey wolves. In addition to the advantages of meta-heuristic algorithms, this algorithm requires no specific input parameters to be initialized. Social dominant hierarchy is employed with the help of alpha (α), beta (β), delta (δ) and omega (ω) wolves as shown in Fig. 2 with alpha (α) wolf as the leader wolf [6].

Group hunting is another interesting social behavior of grey wolves in addition to the social hierarchy of wolves. The three phases of grey wolf hunting are: (i) Tracking, chasing, and approaching the prey, (ii) Pursuing, encircling, and harassing the prey until it stops moving and (iii) Attack towards the prey [6].

![Fig. 2 Hierarchy of grey wolves in GWO algorithm](image)

3.1 Hierarchy in of grey wolves in GWO algorithm

In this section, the mathematical modeling of GWO algorithm is discussed using the social hierarchy of wolves and group hunting of prey.

3.2 Social hierarchy

The fittest solution is considered as the leader wolf or alpha (α) wolf, the second best solution is the beta (β) wolf, the third best solution is the delta (δ) wolf and the rest of the solutions are omega (ω) wolves [6].

3.3 Encircling the prey

As mentioned earlier, the grey wolves encircle the prey during the hunting process. The following two equations are considered to model the encircling behavior of grey wolves [6]:

$$\vec{C} = \vec{B} \times \vec{X}_P(t) - \vec{X}(t)$$ (6)

$$\vec{X}(t + 1) = \vec{X}_P(t) - \vec{A} \times \vec{C}$$ (7)

where $t$ indicates the current iteration in the problem, $\vec{X}_P(t)$ is the position vector of the prey, $\vec{X}(t)$ indicates the position vector of the grey wolf and the coefficient vectors $\vec{A}$ and $\vec{B}$ are computed using the following equations:

$$\vec{A} = 2 \alpha \cdot \vec{r}_1 - \alpha$$ (8)

$$\vec{B} = 2 \vec{r}_2$$ (9)

where $\vec{r}_1$ and $\vec{r}_2$ are random vectors between [0, 1] and $\alpha$ is linearly decreasing from 2 to 0 over each iteration of the problem. Inside the solution space, a grey wolf can update its position in and around the prey in any random location using Eqs. (6) and (7). The same concept can be extended to a search space with $n$ dimensions.

3.4 Hunting

All the grey wolves can recognize the location of prey to encircle and hunt them. The positions of the wolves are updated around the prey using the following equations [6]:

$$\vec{x}_1(t) = \vec{x}_a(t) - \vec{A}_1 \times \vec{C}_a$$

$$\vec{x}_2(t) = \vec{x}_b(t) - \vec{A}_2 \times \vec{C}_b$$

$$\vec{x}(t) = \vec{x}_a(t) - \vec{A}_3 \times \vec{C}_\delta$$ (11)
\[ \vec{X}(t+1) = \frac{\vec{X}_1(t) + \vec{X}_2(t) + \vec{X}_3(t)}{3} \]  

(12)

where \( \vec{X}_a(t) \), \( \vec{X}_\beta(t) \) and \( \vec{X}_\delta(t) \) are the position of first, second and third best fitness value, \( C_a, C_\beta \) and \( C_\delta \) is determined using Eqn. (10), \( A_1, A_2 \) and \( A_3 \) is determined using Eqn. (8) and \( B_1, B_2 \) and \( B_3 \) is determined using Eqn. (9).

Using the above equations, the alpha, beta and delta grey wolves estimates the positions of the prey and other grey wolves randomly update their positions around the prey.

### 3.5 Attacking prey (Exploitation)

Pathway for the grey wolves to approach the prey is mathematically expressed using the parameters \( a \) and \( A \). The parameter \( a \) is linearly decreasing from 2 to 0 and fluctuation of parameter \( A \) also gets decreased with \( \alpha \) [6].

### 3.6 Searching the prey (Exploration)

In general, the wolves diverge from each other to search the prey and converge together to attack the prey. This exploration behavior are influenced directly using the parameters \( \beta \) and \( B \).

Mathematically, when \( \beta \) is greater than 1 or less than -1, the three grey wolves diverge in search of prey. Therefore, from the above discussion we can able to conclude that GWO algorithm shows more random behavior which favors the exploration and avoidance of local optima. The flowchart of GWO algorithm is shown in Fig. 3.

### 4. Implementation of GWO algorithm to ELD (GWO-ELD) problem

The implementation of GWO algorithm to solve ELD complex problem with VPL is described as follows:

**Step 1**: The input data for the chosen test system is read to compute the total fuel cost of the system.

**Step 2**: GWO parameters i.e., population size \( N \) and select the stopping criteria are initialized.

**Step 3**: The number of design variables, \( D \) required for the test system is selected and are initialized. According to the population size, the design variable for the test system is generated randomly using Eqn. (13).

\[ P_{ij} = P_{i_{min}} + \text{rand}(1) \times (P_{i_{max}} - P_{i_{min}}) \]  

(13)

where \( j = 1,2, ..., N \), \( i = 1,2, ..., D \)

Therefore, the matrix of \( D \times N \) is initialized using Eqn.(13).

**Step 4**: The fitness of each population is calculated using \( F_T \). After sorting the fitness value in descending order, alpha (\( \alpha \)), beta (\( \beta \)) and delta (\( \delta \)) grey wolves are determined using Eqn. (14).

\[ F_{T\alpha} = F_T(N), F_{T\beta} = F_T(N-1) \text{and} F_{T\delta} = F_T(N-2) \]  

(14)

**Step 5**: The design variables corresponding to \( F_{T\alpha} \), \( F_{T\beta} \) and \( F_{T\delta} \) are saved as \( \vec{P}_a(t) \), \( \vec{P}_\beta(t) \) and \( \vec{P}_\delta(t) \) respectively.

**Step 6**: Using Eqn. (8) and Eqn. (9), \( A_i \) and \( B_i \), \( i = 1,2,3 \) are determined

**Step 7**: The position of each grey wolf in the population gets updated using Eqn. (10) to Eqn. (12).

**Step 8**: Proper termination criterion is selected

Step 4 to step 7 will be repeated till the termination criteria is reached by the algorithm.

### 5. Numerical results and analysis

The performance analysis of GWO in solving complex ELD problem with VPL is discussed in this section. Standard ELD test systems with 13 and 40 generating units is used for implementing GWO algorithm. Ramp rate limits and individual generator limits are the considered constraints. The GWO algorithm for the test systems has been implemented in MATLAB 2013a on Intel (R) Core (TM) i7 - 3517U CPU 2.40 GHz with 8G-RAM. Result reported in the recent literatures is used for comparing the GWO algorithm in terms of solution quality, robustness and convergence.

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**Test system 1 - 13 unit system**

The first system to be implemented using GWO algorithm with valve point effects is a 13 generator system with 1800 MW as the demand of the system. It is a larger system with more nonlinearities and required system characteristics to determine the total generation cost can be found in [19]. Transmission loss and prohibited operating zones are neglected. Table 1 gives the optimal dispatch solutions obtained by GWO algorithm with a demand of 1800 MW. In an literature survey, the best generation cost reported is 17963.76 $/hr in [20]. The minimum cost obtained by GWO algorithm is 17963.18 $/hr, which is the best generation cost found so far.
The minimum cost achieved by the GWO algorithm is compared with those available in the literatures as summarized in Table 2. Superiority of GWO algorithm over other algorithms can be inferred from Table 2 even though the obtained minimum cost is not guaranteed to be the global solution for the system.

Table 2 Comparison of various algorithms with GWO for test system 1

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Minimum</th>
<th>Average</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSFLA &amp; GA[22]</td>
<td>18954.27</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>CEP [23]</td>
<td>18048.21</td>
<td>18190.32</td>
<td>18400.04</td>
</tr>
<tr>
<td>SA [24]</td>
<td>18084.17</td>
<td>18173.73</td>
<td>18319.39</td>
</tr>
<tr>
<td>PSO [23]</td>
<td>18030.72</td>
<td>18205.78</td>
<td>NA</td>
</tr>
<tr>
<td>MFE[23]</td>
<td>18028.09</td>
<td>18192</td>
<td>18416.89</td>
</tr>
<tr>
<td>FLP[25]</td>
<td>18018</td>
<td>18200.79</td>
<td>18453.82</td>
</tr>
<tr>
<td>EPSQP[22]</td>
<td>17991.03</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>HDE[23]</td>
<td>17975.73</td>
<td>18134.8</td>
<td>NA</td>
</tr>
<tr>
<td>GA[26]</td>
<td>17975.34</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>CGA-MU[23]</td>
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<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>MPSO[27]</td>
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<td>NA</td>
<td>NA</td>
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<tr>
<td>BLBO[24]</td>
<td>17972.86</td>
<td>18015.38</td>
<td>18113.58</td>
</tr>
<tr>
<td>CTLBO [24]</td>
<td>17972.81</td>
<td>18080.87</td>
<td>18243.12</td>
</tr>
<tr>
<td>RTO[23]</td>
<td>17969.8024</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>PSM[23]</td>
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<tr>
<td>UHG[23]</td>
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<td>NA</td>
</tr>
<tr>
<td>CSA[30]</td>
<td>17963.83</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>ORCSA[30]</td>
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<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>OWO[22]</td>
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<td>NA</td>
</tr>
<tr>
<td>MABC[29]</td>
<td>17963.83</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>SDE[20]</td>
<td>17963.82</td>
<td>17963.97</td>
<td>17964.09</td>
</tr>
<tr>
<td>SSA[20]</td>
<td>17963.76</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>GWO</td>
<td>17963.18</td>
<td>17963.89</td>
<td>17965.34</td>
</tr>
</tbody>
</table>

5.4.2 Test system 2 - 40 unit system

Finally, to test the scalability and efficiency of the GWO algorithm it is implemented to a large system with 40 generating units with VPL effects. The total demand of the system is 10500 MW. The system data to determine the generation cost is considered from [19]. After having a literature survey over the state-of-the-art algorithms in solving this system, the best generation cost achieved is 121412.54$/hr and is reported in [21]. But the optimal generation cost achieved by GWO algorithm is 121411.11$/hr. The optimal generation results obtained using GWO algorithm is summarized in Table 3.

Table 3 Optimal power for 40 unit system using VPL with GWO

<table>
<thead>
<tr>
<th>Unit</th>
<th>Real power output in MW</th>
<th>Unit</th>
<th>Real power output in MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>110.7999</td>
<td>P2</td>
<td>523.27937</td>
</tr>
<tr>
<td>P2</td>
<td>110.7998</td>
<td>P3</td>
<td>523.27937</td>
</tr>
</tbody>
</table>

5.1 RESULT ANALYSIS

5.1.1 Parameter selection

The efficiency of stochastic search algorithms such as GA, PSO, DE, etc. depends on user defined parameters. In general, for a specific test system, parameter tuning, testing and evaluation steps are followed to obtain the optimal parameter values of an algorithm [33][34]. Selection of grey wolf population in GWO affects the convergence and search capability of the algorithm. An optimal choice
of population size is necessary as large value makes an algorithm slow and computationally inefficient and small value leads to local minima than to the global minima. The optimal population size directly depends on problem dimension and complexity to achieve optimum value for the problem [35][36].

Tables 2 and 4 provides the required evidence that the optimal fuel cost obtained using GWO algorithm is minimum when compared to the recent literatures. In order to select the optimal population, the performance of GWO algorithm when executed for 50 test runs for different population sizes are presented in Table 5. This proper tuning of parameters steers the GWO algorithm towards global optimal solution making it more efficient, robust and consistent.

<table>
<thead>
<tr>
<th>Table 5. Effect of population size on different test systems</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N</strong></td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>SYS1: 13 unit system</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>70</td>
</tr>
<tr>
<td>90</td>
</tr>
<tr>
<td>SYS2: 40 unit system</td>
</tr>
<tr>
<td>100</td>
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<tr>
<td>120</td>
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<tr>
<td>140</td>
</tr>
<tr>
<td>160</td>
</tr>
<tr>
<td>180</td>
</tr>
</tbody>
</table>

5.4.2 Convergence characteristics

Figure 4 presents the convergence characteristic of GWO algorithm for the discussed two test systems. The algorithm attains the optimal solution within an admissible number of iterations for test systems due to (i) promising regions of the solution space are explored quickly by the search agents and (ii) algorithm has good computational and search mechanism.

5.4.3 Robustness

In general, all the stochastic optimization algorithm inherits random property. Hence, the performance of such algorithms cannot be judged through a single trial. Similarly, the performance and strength of GWO, a recent stochastic algorithm is tested through 100 trials with different initial population values for the chosen test system. The robustness or the consistency level of the GWO algorithm for the chosen test system is shown in Fig. 5.

5.4.4 Superiority of GWO algorithm

Adaptive parameters \( A \) and \( B \) In the mathematical formulation of GWO, reveals that half of the generations is allotted to exploration \( (|A|>1) \) and the rest to the exploitation \( (|A|<1) \). This type of allocation promotes the exploration of the solution space which leads in determining the diverse solution space during algorithm process. Also, the parameter \( B \) which randomly forces the search agents to take random steps towards the optimum solution which is very helpful in resolving the local optima problem. These adaptive parameters which smoothly balance the solution space between the exploration and exploitation is the main reason for the success of GWO algorithm. In addition, in GWO in every generation the best  three solutions are saved which also guides the search agents to exploit the most promising regions of solution space. These are the reasons which assist the GWO algorithm to provide
good exploration, exploitation, local optima avoidance and fast convergence simultaneously. These performances of GWO algorithm on the ELD problems with and without valve-point loading with or without all the constraints conveys that the GWO algorithm can be successfully applied to different types of practical power system optimization problems in future.

6. CONCLUSION

This paper presents a novel GWO algorithm in solving non-convex and convex ELD problems with VPL. The effectiveness, feasibility and robustness of the GWO algorithm has been investigated on ELD problems with VPL with 13 and 40 generating units. Tests were carried on systems with different kinds of constraints. The simulation results show that the proposed algorithm has succeeded in achieving minimum fuel cost and the statistical results were compared with recent results reported in the literature. The success of GWO algorithm on the test system illustrates the efficiency and robustness of the GWO algorithm in solving ELD problems. In future, a study can be taken on dynamic ELD problems and the multi-objective ELD problems considering environmental impact and to implement the successful GWO algorithm to it.

REFERENCES


