Research Paper
EVALUATION OF THE CAPACITANCE OF UNIT CUBE AND CONDUCTING BODIES FOR SURFACE CHARGING ANALYSIS

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ABSTRACT
A numerical analysis for computation of free space capacitance of different arbitrarily shaped conducting bodies based on the finite element method with triangular elements modeling is presented. Evaluation of capacitance of different arbitrary shapes is important for the electrostatic analysis. Capacitance computation is an important step in the prediction of electrostatic discharge which causes electromagnetic interference. We specifically illustrated capacitance computation of three electrostatic models like unit cube, L-shaped plate and a small cube on top of the large cube. Numerical data on capacitance of conducting objects are presented. The results are compared with other available results in the literature. We used the COMSOL multiphysics software for simulation. The models are designed in three-dimensional form using electrostatic environment and can be applied to any practical design. The findings of this study show that the finite element method is a more accurate method for the computation of electrical capacitance.

KEYWORDS: Capacitance, spacecraft circuit modeling, electrostatic analysis, finite element method, electrostatic discharge.

I. INTRODUCTION
The calculation of electrical capacitance of different arbitrary shapes like unit cube, L-shaped plate and a small cube on top of the large cube, which can be considered as significant objects for spacecraft surface charging design. The external surface of the spacecraft design depends on how efficiently a physical structure has been modeled. A well-designed model not only enables conducting a potential study but also reduces the number of iterations associated with the model. This study gives a complete insight into the properties of devices and circuits including transmission, emission, electrostatic effects, etc. the problems related to electromagnetic field do not have a systematic solution, and a mathematical approach is essential. In addition, the studies involving electromagnetic field are usually complex and require a very good working knowledge.

The evaluation of capacitance of different arbitrary shapes is important in computational electromagnetics (CEM). It deals with the modeling of the interaction between the electromagnetic fields and the physical objects [1-2]. Compared to the finite difference methods (FDM) and boundary element methods (BEM), the finite element methods provide additional elasticity for local mesh refinement, additional rigorous convergence analysis, additional selections of effective iterative solvers for the secondary linear systems and more elasticity for handling the nonlinear equations. The FEM [1-3] is a standard tool for solving the differential equations in electromagnetics. It is also one of the most preferred methods in engineering owing to its significant ability to deal with complex geometries.

In this paper, the capacitance of the different geometrical assemblies was achieved by subdividing the structure into triangular subsections. The disadvantage of rectangular subsections is that it will not exactly fit into the any arbitrarily shaped geometry. In order to avoid the disadvantage, triangular patch modeling had been in use to perfectly modeling the arbitrarily shaped surfaces encountered in practical situations [4-6]. The FEM is a simpler and easier method compared to other techniques. This method is suitable for solving differential equations and utilizes a more powerful and useful numerical technique for handling the electromagnetic analysis including difficult geometries [7-9] and inhomogeneous media. However, an accurate understanding of the computed results [10-12] is essential. It is more important to ensure that the implemented models can be applied to the actual problem to be solved, and the results can be obtained with sufficient speed and accuracy.

2. METHOD
2.1. Expression of capacitance
The FEM is a simpler and easier method compared to other techniques. This method is suitable for solving differential equations and utilizes a more powerful and useful numerical technique for handling the electromagnetic analysis including difficult geometries and inhomogeneous media. An efficient and exact computer model of various electromagnetic field problems, including spacecrafts, is made possible using modern high-speed computers and well-developed mathematical techniques. This model enables an spacecraft designer to visualize the targeted spacecraft on the desktop, thereby providing more information in many cases than can ever be measured in the laboratory. The turn-around time required to obtain the spacecraft properties after varying the spacecraft shape is usually calculated in minutes or hours by computer model. The designer can adjust the spacecraft by modifying certain specific parameters of the simulation model. The precision of the existing mathematical model is often such that only a small degree of adjustment is required. However, an accurate understanding of the computed results is essential. It is more important to ensure that the implemented models can be applied to the actual problem to be solved, and the results can be obtained with sufficient speed and accuracy [7].

FEM is well suitable for arbitrary shapes. The simple model of the FEM is based on the behavior of a function, which may be complex when viewed from an enormous region while a simple evaluation may be appropriate for a minor subregion. The entire region is separated into non overlapping subregions called as finite element, and the function of each element is approximated by the algebraic expression [8]. In
addition, the algebraic representations provide continuity of the function. The efficient
generalization of the method makes it possible to
build general-purpose computer programs for solving
a wide range of difficulties.
The expression of capacitance can be introduced by
applying the FEM, i.e., charges and potentials in any
system of conductors that create an electric field.
Depending on the nature of the system of conductors
measured, the capacitance of a solitary conductor, the
capacitance between two conductors, and the capacitance in a system of many conductors can be distinguished. The capacitance of the surface can be computed from
\[ C = Q / V \]  
(1)
Where, \( Q \) is the charge of the conductor (Coulomb).
\( V \) is the potential of the conductor (Voltage).
Calculating the capacitance of a simple system like a
sphere is denoted by the following expression:
\[ C = 4 \pi \varepsilon_0 a \]  
(2)
Where, 
\( \varepsilon_0 \) is the permittivity of free space (=8.854 \times 10^{-12} \text{ F/m})
a is the radius of the sphere (meter)
The potential \( V \) is calculated by
\[ V(x,y) = a + bx + cy \]  
(3)
For a triangular element, \( a, b, \) and \( c \) are constants.
A typical triangular finite element for putting in
place of the equation governing the element.
\[ V_1 = a + bx_1 + cy_1 \]  
(4)
\[ V_2 = a + bx_2 + cy_2 \]  
(5)
\[ V_3 = a + bx_3 + cy_3 \]  
(6)
Similar transformation matrices can be achieved
when higher order plane elements are used. The
shape functions are calculated in truss element nodes
with known coordinates. Typically these coordinates
are agreed in the global coordinates while the shape
functions of the plane elements are agreed in the
natural coordinate systems. The surface to be
analyzed is divided into \( N \) number of triangular
subsections. The geometry of the reference element is
mapped into a global coordinates of \( x \) and \( y \) as shown
in the Figure 1.

**Figure 1**: Mapping of source triangle into three subtriaingles
For triangular elements, the global coordinates \((x, y)\) and
the natural coordinates \((\xi, \eta)\) are given by
\[ x = N_1 x_1 + N_2 x_2 + N_3 x_3 \]  
(7)
\[ y = N_1 y_1 + N_2 y_2 + N_3 y_3 \]  
(8)
where \( N_1, N_2 \) and \( N_3 \) are the geometrical
transformation function, in which \( x_i, y_i \) \( (i=1,2,3) \)
represents the coordinates of the triangular element
nodes and the shape functions have the following
expressions represented in the natural coordinate
system as:
\[ N_1 = \xi, \quad N_2 = \eta, \quad N_3 = 1 - \xi - \eta \]  
(9)
For the known \( x, y \) coordinates, the corresponding
natural coordinates can be obtained by solving the
following system of equations:
\[ \begin{bmatrix} x - x_1 \\ y - y_1 \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \]  
(10)
Similarly, it is possible to obtain the natural
coordinates in the case when the plane element is
square.
The Jacobian of the transformation is written as
\[ J(\xi, \eta) = \begin{vmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_1}{\partial \eta} \\ \frac{\partial N_2}{\partial \xi} & \frac{\partial N_2}{\partial \eta} \end{vmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \]  
(11)
The coordinate transformation for subtriangle 1 is
obtained as
\[ \xi^1 = \frac{1 + \xi}{2}; \quad \eta^1 = \frac{(1 - \xi)(1 + \eta)}{4} \]  
(12)
The transformation for subtriangle 2 is given by
\[ \xi^2 = \frac{1 + \xi}{2}; \quad \eta^2 = \frac{(1 - \xi)(1 + \eta)}{4} \]  
(13)
The transformation for subtriangle 3 is
\[ \xi^3 = \frac{1 + \xi}{2}; \quad \eta^3 = \frac{(1 - \xi)(1 + \eta)}{4} \]  
(14)
The final Jacobian is given as
\[ J(\xi, \eta) = J(\xi, \eta) \begin{vmatrix} \frac{1}{3} \left(1 - \xi^2\right) \end{vmatrix} \]  
(15)
Using the above expressions, free space
capacity can be calculated \([3, 4]\) and spacecraft
potential can be predicted accurately. The
expressions are compact. It is easily fit for the
simulation software. However, the algorithm takes
more time since the model is more complex.

3. SIMULATION

3.1 Capacitance of unit cube
There is no analytical expression for calculating the
electrical capacitance of a unit cube. In this section,
the finite element method is used for calculating unit
cube \([12, 13]\). Figure 2 shows a unit cube with each
side measuring 1.0 m.
The model is designed in three-dimensional modeling
using electrostatic environment. In the boundary
condition of model design, we used ground boundary
which is zero potential. For the unit cube, the bottom
of the cube is specified as ground with a voltage of 0
V. The top of the cube has a specified voltage of 1 V.
It is assumed that the unit cube is made of highly conductive material in which the total resistive value is much lower. When the unit voltage is applied to the object the charge densities near the edge of the bodies [1, 3] are much higher than those far away from the edges.

From the unit cube model, we produced more number of subsections and 2300 domain element in the finite element mesh shown in Figure 3(a). The potential distribution simulations help to better understand the potential distribution of the metallic object. The modeling produces the finite element mesh with triangular subsections and 1536 boundary elements, which shows that the three-dimensional view of the unit cube with triangular subsections.

In this section, the simulation of the unit cube is analyzed. The findings of this study are in accordance with the available results [6, 18]. According to Wintle [19], a random walk method was used for the computation of unit cube; in which the authors found error in the method of parallel curves. Since a random walk method uses smaller steps, they have been distributed randomly as well as these methods avoid hidden systematic errors. Similarly, Mascagni and Simonov [18] used Monte Carlo technique for the computation of the capacitance of the unit cube. In order to estimate the computational error; Markov chain version of the central limit theorem was used.

### 3.2 Capacitance of L-shaped plate

In this section, the finite element method is used for calculating L-shaped metallic plate is discussed. Figure 4 shows L-shaped plate with a dimension of 2.0 m, 1.0 m, and 3.0 m.

The model is designed in three-dimensional modeling using electrostatic environment. Figure 5(a) shows the triangular subsection of L-shaped plate.

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Method</th>
<th>Capacitance/pF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Van Bladel [20]</td>
<td>Variation method</td>
<td>65.56</td>
</tr>
<tr>
<td>Read [17]</td>
<td>Refined boundary element method</td>
<td>66.07</td>
</tr>
<tr>
<td>Wintle [19]</td>
<td>Random walk method with variance reductions</td>
<td>66.06</td>
</tr>
<tr>
<td>Mascagni [18]</td>
<td>Random walk on the boundary</td>
<td>66.07</td>
</tr>
<tr>
<td>Present method</td>
<td>Proposed method</td>
<td>66.05</td>
</tr>
</tbody>
</table>

In some cases, the capacitance values are calculated with respect to variation in number of domain elements and boundary elements. The results are tabulated in Table 1. The capacitance value 66.05 pF obtained is equated with the value obtained from the earlier result. The results tend to converge, and the deviation in analytical and numerical results decreases. It is clear from the table that the present solver lead to very accurate results. Present method takes less time for simulation.
The potential distribution is constant between the cubes, but variances are seen at the center of the cube. These results obtained are in agreement with the earlier results. The capacitance value of 75.4 pF was obtained for total variety of 5344 triangular subdivisions.

As explained by Chow et al. [11], the point matching method with elastance matrix used for computing the capacitance of a small cube on top of a large cube. This current study demonstrates that the convergence can be obtained for the metallic cube for a finite number of elements. A computer program based on the FEM was simulated to determine the capacitance and charge distribution of a small cube on top of the large cube.

4. RESULTS AND DISCUSSION

In the present study, the capacitance of the unit cube was found to be 66.05 pF, which is similar to capacitance obtained in other studies [6]. Similarly, the capacitance of a T-shaped plate was also computed as 93.93 pF, which is similar to that obtained in other studies, i.e., 94.04pF. Moreover, the capacitance of a small cube on top of a large cube was 75.4 pF, which is similar to that obtained in other studies, i.e., 76.2 pF. The application of numerical techniques involves the usage of computers [24] and appropriate program packages. The outcome of the present study shows that the FEM is more effective and accurate than the other methods.

The results of two geometries are summarized in Table 2 and compared with the earlier results [5, 11]. Using FEM, more accurate value is achieved. The process outlining the usage of triangular subdivision produces more accurate value. All simulations were performed on a PC with core-i3 processor with 3.1 GHz CPU and 8 GB of RAM. COMSOL results for the capacitance of the model compared with the previous work that investigated numerical method. In this study, the values obtained matched with the available results.

Table 2. Comparison of Capacitance of different arbitrary shapes

<table>
<thead>
<tr>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>L-shaped plate</td>
<td>93.93</td>
<td>94.04</td>
</tr>
<tr>
<td>Small cube on top of a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>large cube</td>
<td>75.4</td>
<td>76.2</td>
</tr>
</tbody>
</table>

However, this study has certain limitations that have been acknowledged. The simulation was confined to other methods like random walk method and method of moment. Nevertheless, in spite of these limitations, this study provided new insights into the capacitance computation of different conducting bodies.

5. CONCLUSION

FEM has been found to be most accurate method for evaluating free space capacitances. In the present study, different arbitrary shapes are analysed for electrostatic modeling. The capacitance of different arbitrary shaped conductors like unit cube, L-shaped plate and a small cube on top of a large cube were calculated. Some of the simulations obtained in the study show the usage of FEM with COMSOL multiphysics software. The results derived from using the software correspond with the results of...
previous studies. Thus, the method is more suitable for various shapes involved in spacecraft circuit modelling design. This approach was simple and can be applied to any practical shapes of the metallic objects.

REFERENCES

17. F. H. Read, Capacitances and singularities of the unit triangle, square, tetrahedron and cube, COMPEL - The international journal for computation and mathematics in electrical and electronic engineering23(2) (2004), 572 – 578.
23. M. Dhamodaran and R. Dhanasekaran, Comparison of Computational Electromagnetics for Electrostatic