Research Paper

LABELING OF DOUBLE TRIANGULAR SNAKE

Dushyant Tanna

Address for Correspondence

Department of Mathematics, Marwadi Engineering College, India

ABSTRACT:

The problems arising from the study of a variety of labeling techniques of the elements of a graph or of any discrete structure form one such potential area of challenge. Labeling of graphs is very useful in networking, optimization and some new concepts like Cross – Entropy. Double Triangular snake labeling is earning lots of famine these days.

KEYWORDS: Double Triangular Snake, Harmonious labling, Triangular Cactus, Triangular Snake

Subject Classification code: **05C78**

1. INTRODUCTION

Labeling of double triangular snake is very important in computer networking. The harmonious labeling was introduced by Graham and Sloane [1], during 1980, this labeling was motivated through the study of additive basis in number theory. Here we will present harmonious labeling of Double Triangular Snake investigated by Yue et al [2], Y. Yuansheng, W. Liping [2]

2. DEFINITIONS

2.1 Harmonious Labeling

Let G be a graph with q edges. A function f is called harmonious labeling of graph G if f: V $\rightarrow$ {0, 1, 2,,,,q-1} is injective and the induced function f*: E $\rightarrow$ {0, 1, 2,,,,,,q} defined as f*(e = uv) = (f(u) + f(v))(mod q) is bijective. A Graph which admits harmonious labeling is called harmonious graph

2.2 Triangular Cactus

A Triangular Cactus is a connected graph all of whose blocks are triangles

2.3 Triangular Snake

A triangular snake is a triangular cactus whose block – cut point – graph is a path, i.e. a triangular snake with p triangles is obtained from a path v_0, v_1, ,,,, v_p by joining v_i and v_{i+1} to a new vertex w_i for i = 0,1,,,, p-1.

2.4 Double Triangular Snake

A double triangular snake is a graph formed by two triangular snake having a common path, i.e. a double triangular snake with p blocks is obtained from a path v_0, v_1,,,, v_p by joining v_i and v_{i+1} to two new vertices v_{p+1+i} and v_{2p+1+i} for i = 0,1,,,,,p-1.

3. THEOREM

All the double triangular snakes are harmonious

Proof:

Let Gp be a double triangular snake with p blocks on n vertices and q edges. Than n = 3p + 1 and q = 5p.

Let

$V_1 = \{v_0, v_1,,,, v_p\}$,

$V_2 = \{v_{p+1}, v_{p+2},,,,v_{2p}\}$,

$V_3 = \{v_{2p+1}, v_{2p+2},,,,v_{3p}\}$,

$E_1 = \{v_iv_{i+1} : 0 \le i \le p-1\}$,

$E_2 = \{v_iv_{p+1+i} : 0 \le i \le p-1\}$,

$E_3 = \{v_{i+1}v_{p+1+i} : 0 \le i \le p-1\}$,

$E_4 = \{v_iv_{2p+1+i} : 0 \le i \le p-1\}$,

$E_5 = \{v_{i+1}v_{2p+1+i} : 0 \le i \le p-1\}$,

$V(Gp) = V_1 \cup V_2 \cup V_3$.

$E(Gp) = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5$

$Sk = \{ f(v_i) : v_i \in Vk\}$, $1 \le k \le 3$,

$Dk = \{ g(v_iv_j) : v_iv_j \in Ek\}$, $1 \le k \le 5$.

Case 1: p ≡ 1 (mod 2)

We label the vertices of V(Gp) as follows:

$$f(v_i) = \begin{cases} i, & 0 \le i \le (p-1)/2 \\ 2p-1+i, & (p+1)/2 \le i \le 3p \end{cases}$$

Than we have,

$S_1 = \{f(v_i) / 0 \le i \le p\}$

$= S_{11} \cup S_{12}$.

$S_{11} = \{f(v_i) / 0 \le i \le (p-1)/2\} = \{i / 0 \le i \le (p-1)/2\}$

$= \{0,1,,,,(p-1)/2\}$,

$S_{12} = \{f(v_i) / (p+1)/2 \le i \le p\} = \{2p-1+i / (p+1)/2 \le i \le p\}$

$= \{(5p-1)/2, (5p+1)/2,,,,,3p-1\}$,

$S_2 = \{f(v_i) / p+1 \le i \le 2p\} = \{2p-1+i / p+1 \le i \le 2p\}$

$= \{3p, 3p+1,,,,,4p-1\}$,

$S_3 = \{f(v_i) / 2p+1 \le i \le 3p\} = \{2p-1+i / 2p+1 \le i \le 3p\}$

$= \{4p, 4p+1,,,,,5p-1\}$.

Hence,

$S = S_1 \cup S_2 \cup S_3$

= S_{11} U S_{12} U S_2 U S_3
= {0,1,,,,(p-1)/2} U {(5p-1)/2, (5p+1)/2,,,,,3p-1} U {3p, 3p+1,,,,,4p-1} U {4p, 4p+1,,,,,5p-1}.
= {0,1,,,,(p-1)/2, (5p-1)/2, (5p+1)/2,,,,,,5p-1}

It is obvious that the labels of each vertex are different.
And Max $\{f(v_i) : 0 \le i \le p\} = 5p - 1 \le q - 1$.
So we can say that f: V(Gp) $\rightarrow$ {0,1,2,,,,5p – 1} is injective.
Secondly, it can be easily verified that g maps E(Gp) onto {0,1,2,,,,5p – 1}

D_1 = {g (v_iv_{i+1}) / 0 ≤ i ≤ p} = D_{11}U D_{12} U D_{13},
D_{11} = {g (v_iv_{i+1}) / 0 ≤ i ≤ (p – 3)/2} = {2i +1 / 0 ≤ i ≤ (p-3)/2} = {1,3,,,p-2},
D_{12} = {g (v_iv_{i+1})/ i = (p – 1)/2} = {2p+2i / i = (p-1)/2} = {3p-1},
D_{13} = {g (v_iv_{i+1}) / (p+1)/2 ≤ i ≤ (p – 1)} = {2i–p–1 / (p+1)/2 ≤ i ≤ (p - 1)} = {0,2,,,,,p-3},
D_2 = {g (v_iv_{p+i+1}) / 0 ≤ i ≤ p – 1} = D_{21} U D_{22},
D_{21} = {g (v_iv_{p+i+1})/0 ≤ i ≤ (p – 1)/2} = {3p+2i /0 ≤ i ≤ (p-1)/2} = {(3p, 3p+2,,,4p-1},
D_{22} = {g (v_iv_{p+i+1}) / (p+1)/2 ≤ i ≤ (p – 1)} = {2i–1 / (p+1)/2 ≤ i ≤ (p - 1)} = {p, p+2,,,,,2p-3},
D_3 = {g ($v_{i+1}v_{p+i+1}$) / 0 ≤ i ≤ p – 1} = D_{31} U D_{32},
D_{31} = {g ($v_{i+1}v_{p+i+1}$)/0 ≤ i ≤ (p – 3)/2} = {3p+1+2i /0 ≤ i ≤ (p-3)/2} = {(3p+1, 3p+3,,,4p-2},
D_{32} = {g ($v_{i+1}v_{p+i+1}$) / (p-1)/2 ≤ i ≤ (p – 1)} = {2i / (p-1)/2 ≤ i ≤ (p - 1)} = {p-1, p+1,,,,,2p-2},
D_4 = {g (v_iv_{2p+i+1}) / 0 ≤ i ≤ p – 1} = D_{41} U D_{42},
D_{41} = {g (v_iv_{2p+i+1})/0 ≤ i ≤ (p – 1)/2} = {4p+2i /0 ≤ i ≤ (p-1)/2} = {(4p, 4p+2,,,5p-1},
D_{42} = {g (v_iv_{2p+i+1}) / (p+1)/2 ≤ i ≤ (p – 1)} = {p-1+2i / (p+1)/2 ≤ i ≤ (p - 1)} = {2p, 2p+2,,,,,3p-3},
D_5 = {g ($v_{i+1}v_{2p+i+1}$) / 0 ≤ i ≤ p – 1} = D_{51} U D_{52},
D_{51} = {g ($v_{i+1}v_{2p+i+1}$)/0 ≤ i ≤ (p – 3)/2} = {4p+1+2i /0 ≤ i ≤ (p-3)/2} = {(4p+1, 4p+3..,,,5p-2},
D_{52} = {g ($v_{i+1}v_{2p+i+1}$) / (p-1)/2 ≤ i ≤ (p – 1)} = {p+2i / (p-1)/2 ≤ i ≤ (p - 1)} = {2p-1, 2p+1,,,,,3p-2}.

Hence,
D = D_1 U D_2 U D_3 U D_4 U D_5
= D_{13} U D_{11} U D_{32} U D_{22} U D_{52} U D_{42}U D_{12} U D_{21} U D_{31}U D_{41} U D_{51}
= {0,2,,,,,p-3} U {1,3,,,p-2} U {p-1, p+1,,,,,2p-2} U {p, p+2,,,,,2p-3}U{2p-1, 2p+1,,,,,3p-2}U{2p, 2p+2,,,,,3p-3}
U {3p-1} U {(3p, 3p+2,,,4p-1} U {(3p+1, 3p+3,,,4p-2} U {(4p, 4p+2,,,5p-1} U {(4p+1, 4p+3..,,,5p-2}
= {0,1,2,,,,,5p-2, 5p-1}

Thus, f: E $\rightarrow$ {0,1,2,,,,,5p-1} is bijective as all the edge labels are different.

Case 2: p ≡ 0 (mod 2)

Let

$$\theta = \begin{cases} 2, & p \bmod 8 = 2 \\ 0, & otherwise \end{cases}$$

$$\varepsilon = \begin{cases} 1, & p \bmod 8 = 0 \\ -1, & p \bmod 8 = 4 \\ 0, & otherwise \end{cases}$$

$$\alpha = \begin{cases} 1, & p \bmod 8 = 2,4 \\ 0, & otherwise \end{cases}$$

We label the vertices of V(Gp) as follows:

$$f(v_i) = \begin{cases} \text{i}, & 0 \le i \le p/2, \\ 4p-1+i, & (p/2)+1 \le \text{i} \le \text{p and } i \equiv (p/2)+1 (\text{mod } 2), \\ \text{i}, & (p/2)+2 \le \text{i} \le \text{p and } i \equiv (p/2) (\text{mod } 2), \\ 3p-1+i & p+1 \le i \le 3p/2, \\ (11p/2)+3-2i-\theta & (3p/2)+1 \le \text{i} \le 2p-\alpha \text{ and } i \equiv ((p/2)+1)(\text{mod } 2) \\ 5p+2-2i+\varepsilon & (3p/2)+2 \le \text{i} \le 2\text{p}-\alpha \text{ and } i \equiv (p/2)(\text{mod } 2) \\ 2p+2 & 2p-\alpha \le i \le 2p \; and \; p \bmod 8 = 2 \\ (3p/2)+1, & 2p-\alpha \le i \le 2p \; and \; p \bmod 8 = 4 \\ p+1+i, & 2p+1 \le i \le (5p/2)-1 \\ p+((P+2) \bmod 4)/2, & i = 5p/2 \\ f(v_{i-p})-1, & (5p/2)+1 \le i \le 3p \end{cases}$$

It is easy to verify that the labels of all vertices are different and Max $\{f(v_i): 0 \le i \le 3p\} = 5p - 2 \le q - 1$. So we can say that f: V(Gp) $\rightarrow$ {0,1,2,,,,5p – 1}. And using above pattern g: E $\rightarrow$ {0, 1, 2,,,,,5p – 1} is objective. So we can conclude that the double triangular snakes are harmonious for p ≡ 0 (mod 2).

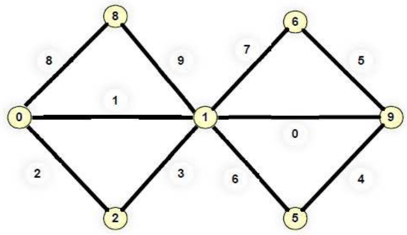
Thus in both the cases described above the graph double triangular snakes admits harmonious labeling.

4. EXAMPLES

We give the following two illustrations with reference of above theorem. We use the following labeling pattern to show that G2 and G3 admit harmonious labeling

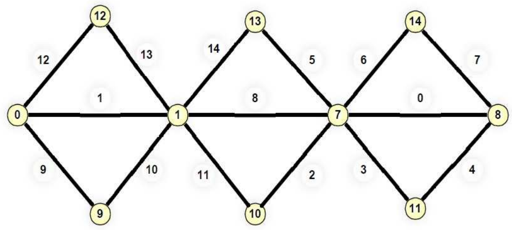
4.1 For G2, we use the following labeling pattern of vertices.

$f(v_0) = 0$, $f(v_1) = 1$, $f(v_2) = 9$, $f(v_3) = 8$, $f(v_4) = 6$, $f(v_5) = 2$, $f(v_6) = 5$.



4.2 For G3 we assign labels of vertices as

$f(v_0) = 0$, $f(v_1) = 1$, $f(v_2) = 7$, $f(v_3) = 8$, $f(v_4) = 9$, $f(v_5) = 10$, $f(v_6) = 11$, $f(v_7) = 12$, $f(v_8) = 13$, $f(v_9) = 14$,



5. CONCLUSION

The harmonious labeling is one of the most important labeling techniques. As all the graphs are not harmonious, it is very interesting to investigate graphs or graph families which admit harmonious labeling. We have reported the harmonious labeling of Double Triangular Snake investigated by Yue et al [2], Y. Yuansheng, W. Liping

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